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Ex : 1

Solution

$f_1(x) = 5x^3 + \frac{2}{3\sqrt{x}} - \frac{5}{x^2} + 3$

$F_1(x) = \frac{5}{4}x^4 + \frac{2}{3}\times 2\sqrt{x} - 5\times\left(-\frac{1}{x}\right) + 3x + k$

$F_1(x) = \frac{5}{4}x^4 + \frac{4}{3}\sqrt{x} + \frac{5}{x} + 3x + k$

est une primitive de f_1 sur]0;+∞[

2°) $F(x) = 2\sqrt{x^3} + \frac{3}{x^5} + 4x - 1$

$F(x) = 2\times x^{3/2} + 3x^{-5} + 4x - 1$

$= 2\times\frac{x^{3/2+1}}{3/2+1} + \frac{3x^{-5+1}}{-5+1} + 4\left(\frac{x^2}{2}\right) - x + k$

$= 2\times\frac{x^{5/2}}{5/2} + \frac{3x^{-4}}{-4} + 2x^2 - x + k$

$= 2\times\frac{2}{5}x^{2+1/2} - \frac{3}{4}\times\frac{1}{x^4} + 2x^2 - x + k$

$F(x) = \frac{4}{5}x^2\sqrt{x} - \frac{3}{4x^4} + 2x^2 - x + k$

est une primitive de f_2 sur]0;+∞[

3°) $f_3(x) = \frac{1}{\cos^2 x} - 7\sin 2x$

$F(x) = \tan x - 7\times\left(-\frac{1}{2}\cos 2x\right) + k$

$F_3(x) = \tan x + \frac{7}{2}\cos 2x + k$

est une primitive de f_3 sur]-π/2 ; π/2[

4°) $f_4(x) = 5x^3(x^4+1)^{2015}$

$F_4(x) = \frac{5}{4}(4x^3)(x^4+1)^{2015}$

$= \frac{5}{4}u'(x)(u(x))^{2015}$

$u(x) = x^4+1$

donc $F_4(x) = \frac{5}{4}\times\frac{(x^4+1)^{2016}}{2015+1} + k$

$F_4(x) = \frac{5}{4}\times\left(\frac{(x^4+1)^{2016}}{2016}\right) + k$

est une primitive de f_4 sur ℝ

5°) $f_5(x) = \tan^{2013}x + \tan^{2015}x$

$= \tan^{2013}x\,(1 + \tan^2 x)$

$u'(x)$ $u(x)$

ou $u'(x) = \tan x$ $n = 2013$

$F_5(x) = \frac{\tan^{2014}x}{2014} + c$

est une primitive de f_5 sur $]-\pi/2 + k\pi \,;\, \pi/2 + k\pi[$ $k \in \mathbb{Z}$

6°) $f_6(x) = \frac{4x^2 - 2x + 1}{x^2(x+1)^2}$

$F_6(x) = \frac{3x^2 + x^2 + 2x + 1}{x^2(x+1)^2}$

$= \frac{3x^2 + (x+1)^2}{x^2(x+1)^2} = \frac{3x^2}{x^2(x+1)^2} + \frac{\cancel{(x+1)^2}}{x^2\cancel{(x+1)}}$

$= \frac{3\cancel{x^2}}{\cancel{x^2}(x+1)^2} + \frac{\cancel{(x+1)^2}}{x^2\cancel{(x+1)^2}}$

$= \frac{3}{(x+1)^2} + \frac{1}{x^2}$

$F_6(x) = \frac{-3}{x+1} - \frac{1}{x} + c$

est une primitive de f_6 sur chacun des http://maurimath.net/Espaceeleves.php

intervalle

$]-\infty;-1[\,;\,]-1;0[\,;\,]0;+\infty[$

7°) $f_7(x) = x^3(x^4+1)^{2015}$

$= \frac{1}{4}(4x^3)(x^4+1)^{2015}$

$= \frac{1}{4}(u'(x)(u(x))^{2015}$

ou $u(x) = x^4+1$ et $n = 2015$

$F_7(x) = \frac{1}{4}\left(\frac{(x^4+1)^{2016}}{2016}\right) + c$

est une primitive de f_7 sur $\mathbb{R}$

8°) $f_8(x) = \cos x\sqrt{1+\sin x}$

$= \cos x(1+\sin x)^{1/2}$

$u'(x)\ (u(x))^{1/2}$

$F_8(x) = \frac{(1+\sin x)^{1/2+1}}{1/2+1} + c$

$= \frac{(1+\sin x)^{3/2}}{3/2} + c$

$F_8(x) = \frac{2}{3}\sqrt{(1+\sin x)^3} + c$

est un primitive de f_8 sur $\mathbb{R}$ (car $1+\sin x \geq 0$ $\forall x \in \mathbb{R}$)

9°) $f_9(x) = \frac{3\sin x}{\sqrt{3+2\cos x}}$

$= -\frac{3}{2} \times \frac{-2\sin x}{\sqrt{3+2\cos x}}$

$= -\frac{3}{2} \times \frac{u'(x)}{\sqrt{u(x)}}$ ou

$u(x) = 3+2\cos x$

$F_9(x) = \frac{-3/2 \times 2\sqrt{3+2\cos x}}{-3}$

$\boxed{F_9(x) = -3\sqrt{3+2\cos x}}$

est une primitive de f_9 sur $\mathbb{R}$ (car $3+2\cos x \geq 0$ $\forall x \in \mathbb{R}$)

10°) $f_{10}(x) = \frac{8x+8}{3\sqrt{x^2+2x+5}}$

$= \frac{4}{3}\left(\frac{2x+2}{\sqrt{x^2+2x+5}}\right)$

$= \frac{4}{3} \times \left(\frac{u'(x)}{\sqrt{u(x)}}\right)$

on $u(x) = x^2+2x+5$

$F_{10}(x) = \frac{4}{3} \times 2\sqrt{x^2+2x+5} + c$

$F_{10}(x) = \frac{8}{3}\sqrt{x^2+2x+5} + c$

est une primitive de f_{10} sur $\mathbb{R}$

car $\forall x \in \mathbb{R}$ x^2+2x+5

$x^2+2x+1+4$

$(x+1)^2+4 > 0$

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Ex: 10

Solution

Methodes

1) changement de bornees
2°) Relation entre differientielle
3°) Remplacement.
4°) Calculer

$$I_1 = \int_1^2 \frac{dx}{(4x+5)^5} \qquad t = 4x+5$$

$\begin{cases} x = 1 \Rightarrow t = 9 \\ x = 2 \Rightarrow t = 13 \end{cases}$

$dt = 4dx \Rightarrow dx = \frac{1}{4} dt$

$$I_1 = \int_9^{13} \frac{1}{4} \cdot \frac{dt}{t^5}$$

$$I_1 = \frac{1}{4} \int_9^{13} t^{-5} dt$$

$$= \frac{1}{4} \left[\frac{1}{-4} t^{-4} \right]_9^{13}$$

$$I_1 = \frac{-1}{16} \left(13^{-4} - 9^{-4} \right)$$

$$I_2 = \int_1^{\sqrt{3}} \frac{dx}{1+x^2} \qquad x = \tan t$$

$x = \tan t \quad t \in [0; \frac{\pi}{2}]$

$\begin{cases} x = 1 \Rightarrow \tan t = 1 \Rightarrow t = \frac{\pi}{4} \\ x = \sqrt{3} \Rightarrow \tan t = \sqrt{3} \Rightarrow t = \frac{\pi}{3} \end{cases}$

$dx = (1 + \tan^2 t)\, dt$

$$I_2 = \int_{\pi/4}^{\pi/3} \frac{\cancel{(1+\tan^2 t)}\, dt}{\cancel{1+\tan^2 t}}$$

$$I_2 = \int_{\pi/4}^{\pi/3} dt = [t]_{\pi/4}^{\pi/3}$$

$$I_2 = \frac{\pi}{3} - \frac{\pi}{4} \Rightarrow \boxed{I_2 = \frac{\pi}{12}}$$

3°) $I_3 = \int_0^{\pi/2} \frac{4\, dx}{1+\cos x} \qquad t = \tan \frac{x}{2}$

$\begin{cases} x = 0 \Rightarrow t = \tan 0 = 0 \\ x = \frac{\pi}{2} \Rightarrow t = \tan \frac{\pi}{4} = 1 \end{cases}$

$dt = \frac{1}{2}\left(1 + \tan^2 \frac{x}{2}\right) dx$

$= \frac{1}{2\cos^2 \frac{x}{2}} dx$

$dx = 2\cos^2 \frac{x}{2}\, dt$

$$I_3 = \int_0^1 \frac{4 \cdot 2\cos^2 \frac{x}{2}}{1 + \cos \frac{x}{2}} dt$$

on sait que
$1 + \cos x = 2\cos^2 x$

Alors: $I_3 = \int_0^1 \frac{4 \cdot \cancel{2\cos^2 x/2}}{\cancel{2\cos^2 \frac{x}{2}}} dt$

$$I_3 = \int_0^1 4dt = [4t]_0^1$$

$$\boxed{I_3 = 4}$$

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4°) $I_4 = \int_2^3 \frac{x^3\,dx}{\sqrt{x-1}}$ $\qquad t = x-1$

$\begin{cases} x=2 \Rightarrow t=1 \\ x=3 \Rightarrow t=2 \end{cases}$

$dt = dx$

$$I_4 = \int_1^2 \frac{(t+1)^3}{\sqrt{t}}\,dt$$

$$I_4 = \int_1^2 \frac{t^3+3t^2+3t+1}{t^{1/2}}\,dt$$

$$= \int_1^2 \left(\frac{t^3}{t^{1/2}} + \frac{3t^2}{t^{1/2}} + \frac{3t}{t^{1/2}} + \frac{1}{t^{1/2}}\right)dt$$

$$= \int_1^2 \left(t^{5/2} + 3t^{3/2} + 3t^{1/2} + t^{-1/2}\right)dt$$

$$= \left[\frac{1}{5/2+1}t^{5/2+1} + \frac{3}{3/2+1}t^{3/2+1} + \frac{3}{1/2+1}t^{1/2+1} + \frac{1}{-1/2+1}t^{-1/2+1}\right]_1^2$$

$$= \left[\frac{2}{7}t^{7/2} + \frac{6}{5}t^{5/2} + 2t^{3/2} + 2t^{1/2}\right]_1^2$$

$$= \left[\left(\frac{2}{7}t^3 + \frac{6}{5}t^2 + 2t + 2\right)\sqrt{t}\right]_1^2$$

$$= \left(\frac{16}{7} + \frac{24}{5} + 6\right)\sqrt{2} - \left(\frac{2}{7} + \frac{6}{5} + 4\right)$$

$$\boxed{I_4 = \left(\frac{458}{35}\right)\sqrt{2} - \frac{192}{35}}$$

5°) $I_5 = \int_0^2 \frac{\sqrt{x+1}}{x^2+3x+2}\,dx$

$t = \sqrt{x+1}$

$\begin{cases} x=0 \Rightarrow t=1 \\ x=2 \Rightarrow t=\sqrt{3} \end{cases}$

$dt = \frac{1}{2\sqrt{x+1}}\,dx$

$\Rightarrow dt = \frac{1}{2t}\,dt$

$dx = 2t \cdot dt.$

$$I_5 = \int_1^{\sqrt{3}} \frac{t}{x^2+3x+2}$$

on constate que

$x^2+3x+2 = (x+1)(x+2)$

$= t^2(t^2+1)$

$$I_5 = \int_1^{\sqrt{3}} \frac{t \times 2t}{t^2(t^2+1)}\,dt$$

$$= \int_1^{\sqrt{3}} \frac{2\,dt}{t^2+1}$$

$t = \tan u$

$t=1 \Rightarrow \tan u = 1 \Rightarrow u = \frac{\pi}{4}$

$t=\sqrt{3} \Rightarrow \tan u = \sqrt{3} \Rightarrow u = \frac{\pi}{3}$

$dt = (1+\tan^2 u)\,du$

$$I_5 = \int_{\pi/4}^{\pi/3} \frac{2(1+\cancel{\tan^2 u})\,du}{\cancel{\tan^2 u}+1}$$

$$= \int_{\pi/3}^{\pi/3} 2\,du$$

$$I_5 = [2u]_{\frac{\pi}{4}}^{\pi/3}$$

$$I_5 = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \Rightarrow \boxed{I_5 = \frac{\pi}{6}}$$

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Exi 4
Solution

on a pour tout $x \in \mathbb{R}$

$$f(x) = x + \sqrt{x^2+1}$$

$$f'(x) = 1 + \frac{2x}{2\sqrt{x^2+1}}$$

$$f'(x) = \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

l'integrale

$$I = \int_0^1 \frac{\left(x+\sqrt{x^2+1}\right)^2}{\sqrt{x^2+1}}\,dx$$

peut être sous la forme

$$I = \int_0^1 \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}} \left(x+\sqrt{x^2+1}\right) dx$$

D'où $I = \left[\frac{1}{2}\left(x+\sqrt{x^2+1}\right)^2\right]_0^1$

$$I = \frac{1}{2}\left(1+\sqrt{2}\right)^2 - \frac{1}{2}(2)^2$$

Enfin $I = \frac{-1+2\sqrt{2}}{2}$

l'integrale

$$J = \int_0^1 \frac{1}{\left(x+\sqrt{x^2+1}\right)\sqrt{x^2+1}}\,dx$$

peut être sous la forme

$$J = \int_0^1 \frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}} \times \frac{1}{\left(x+\sqrt{x^2+1}\right)^2}\,dx$$

d'où

$$J = \left[\frac{-1}{x+\sqrt{x^2+1}}\right]_0^1$$

$$J = \frac{-1}{1+\sqrt{2}} + 1.$$

Enfin $\boxed{J = \frac{\sqrt{2}}{1+\sqrt{2}}}$

Ex : 7

Solution

1°) pour montrer que f est une fonction affine
$f'(x) = cte$ (constante)

le regle :
(Remarque)

$$F(x) = \int_{u(x)}^{v(x)} f(t)\,dt$$

$$\Rightarrow F'(x) = v'(x) \times f(v(x)) - u'(x) \times f(u(x))$$

$$f'(x) = \cos x \sqrt{1-\sin^2 x} + \sin x \sqrt{1-\cos^2 x}$$

$$= \cos x \cdot \cos x + \sin x \cdot \sin x$$

$$= \cos^2 x + \sin^2 x$$

$f'(x) = 1 \Rightarrow f(x) = x + b$

donc : f est affine

2°) Pour calculer b on cherche l'image de 0 par la fonction

$$f(0) = \int_{\cos 0}^{\sin 0} \sqrt{1-t^2}\,dt$$

$$= \int_1^0 \sqrt{1-t^2}\,dt$$

$$= -\int_0^1 \sqrt{1-t^2}\,dt$$

$$= \left[\frac{2}{3}(1-t^2)\sqrt{1-t^2}\right]_0^1$$

$$= -\left(\frac{2}{3}\right) = \frac{2}{3} = b$$

$$\Rightarrow \boxed{f(x) = x + \frac{2}{3}}$$